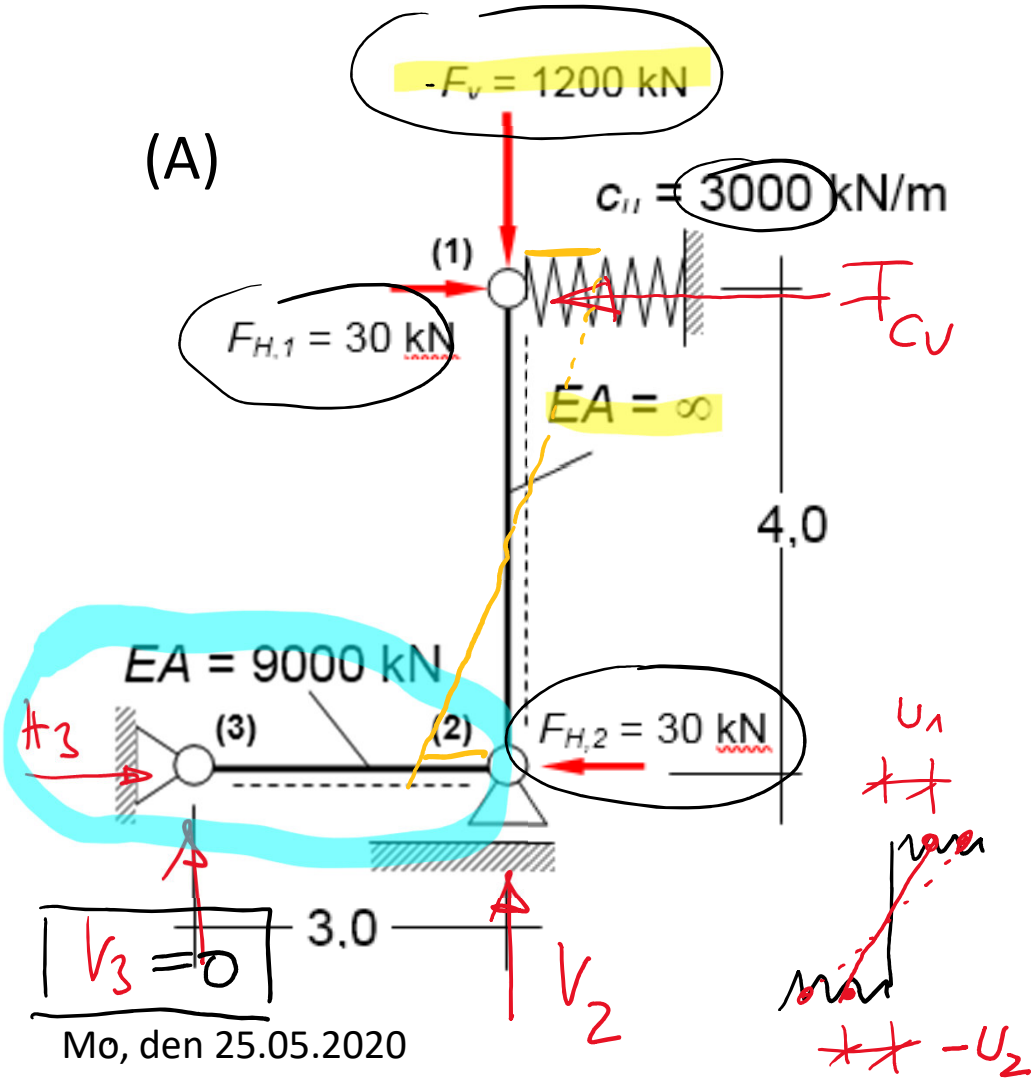
Übungsbeispiele 10: Berechnung nach Theo. II Ordnung

(A)



Zustandsgrößen nach Th. I. O.

$V_2 = 1200\ kN$; $N_{12} = -1200\ kN$

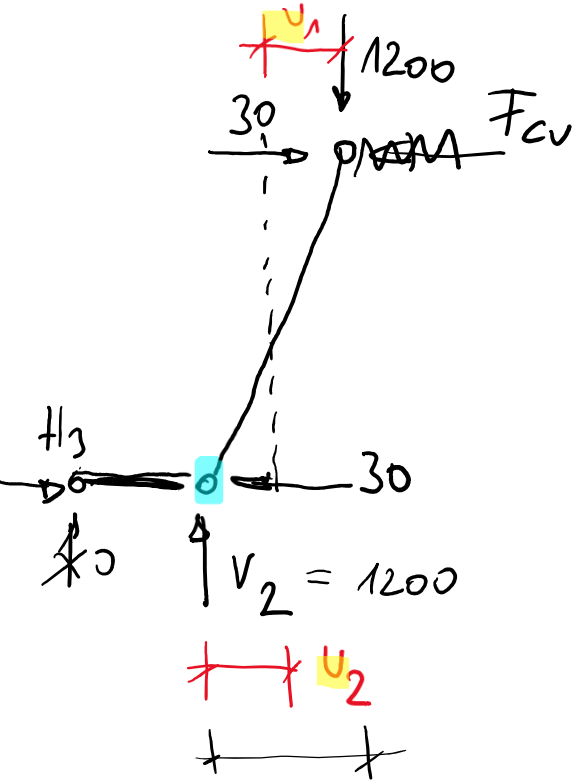
$F_{cu} = 30\ kN$ $N_{32} = -30\ kN$

$H_3 = 30\ kN$

Zugeh. Verformungen:

$$u_1 = \frac{F_{cu}}{c_u} = \frac{30}{3000} = 0{,}01\,m \quad (\text{nach rechts})$$

$$u_2 = \frac{N}{EA/\ell} = \frac{-30 \cdot 3}{9000} = -0{,}01\,m \quad (\text{nach links})$$



1. Iteration:

$\curvearrowleft \sum M_{2,ots} \doteq 0$: $F_{cu} \cdot 4 - 30 \cdot 4 - 1200 \cdot 2 \cdot 0{,}01 \doteq 0$

$F_{cu} = 36\ kN$

$v_{1,neu} = \frac{36}{3000} = 0{,}012\ m$

$\rightarrow \sum H \doteq 0$: $H_3 = 36\ kN$; $N_{32} = -36\ kN$

$v_{2,neu} = \frac{-36 \cdot 3}{9000} = -0{,}012\ m$

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2. Iteration: $(F_{cu} - 30) \cdot 4 - 1200 \cdot 2 \cdot 0{,}012 \doteq 0$

$F_{cu} = 37{,}2\ kN$ $\quad v_{1,neu} = \frac{37{,}2}{3000} = 0{,}0124\ m$

$H_3 = 37{,}2\ kN$ $\quad v_{2,neu} = -0{,}0124\ m$

Aufgabe analyt. berech. mit $v = v_1 = -v_2$ als Unbekannte

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3. Iteration

$$(F_{cu} - 30) \cdot 4 + 1200 \cdot 2 \cdot 0{,}0124 \overset{!}{=} 0 \qquad \boxed{\Delta \to 0}$$

$$F_{cu} = 37{,}5\ kN \quad ; \quad v_{1,neu} = \frac{37{,}5}{3000} = 0{,}0125\ m$$

$$H_3 = 37{,}5\ kN \quad ; \quad v_{2,neu} = -0{,}0125\ m$$

Zustandsgr. nach Th. II. O.

$$F_{cu}^{II} = 37{,}5\ kN \quad ; \quad H_3^{II} = 37{,}5\ kN \rightsquigarrow N_{32}^{II} = -37{,}5\ kN$$

$$V_2^{II} = 1200\ kN \quad ; \quad N_{12}^{II} = -\sqrt{1200^2 + 37{,}5^2}$$

$$= -1200{,}6\ kN$$

$$\text{mit } v_1^{II} = -v_2^{II} = 0{,}0125\ m$$

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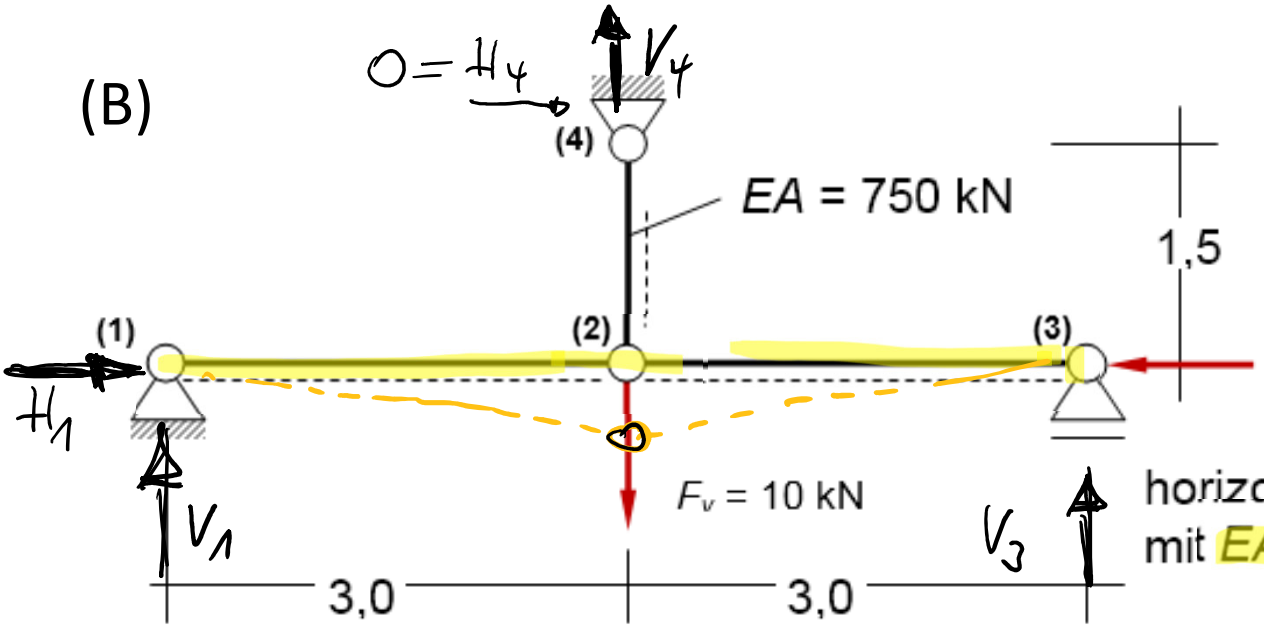
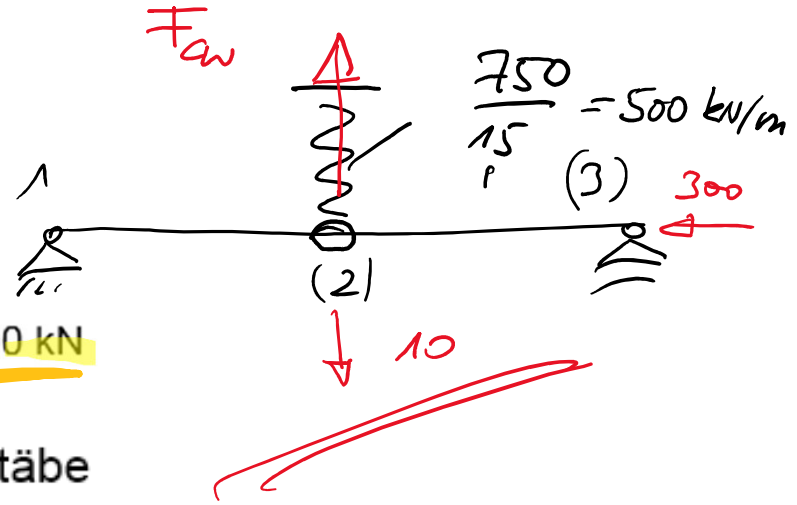
(B)





Zustandsgrößen nach Theo I. O.

$V_1 = 0$; $V_3 = 0$; $H_1 = 300$ kN ; $V_4 = 10$ kN ;

$N_{24} = +10$ kN

$N_{12} = N_{23} = -300$ kN

zugehörige Verformung: $w_2 = ? = \frac{10}{500} = 0{,}02$ m

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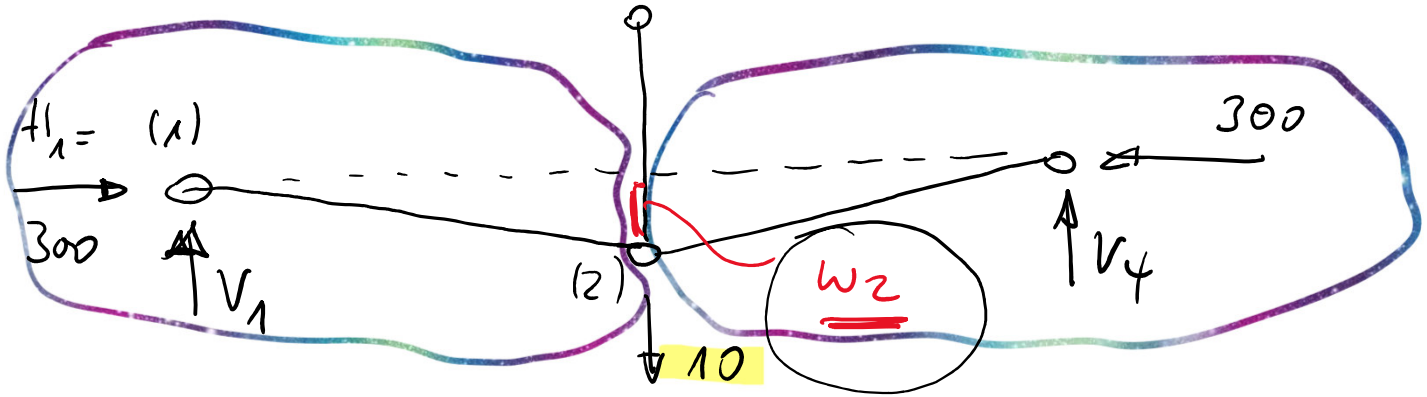
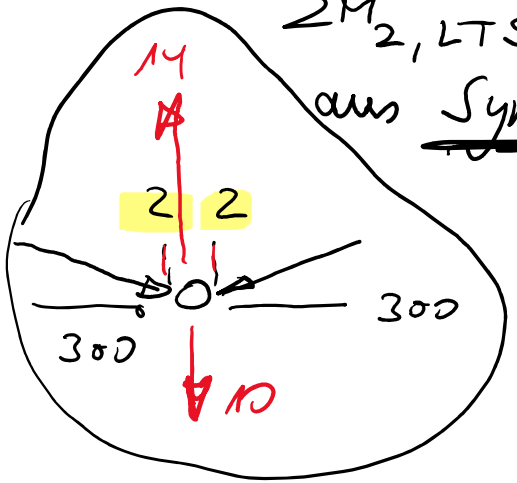


1. Iteration: $H_1 = 300$ kN:

$\Sigma M_{2,LTS} = 0$: $-V_1 \cdot 3.0 - 300 \cdot 0{,}02 = 0$ $\rightsquigarrow$ $V_1 = -2{,}0$ kN

aus Symmetrie: $V_4 = -2{,}0$ kN $\rightsquigarrow$ $V_4 = 10 + 2 + 2 = 14$ kN



$N_{42} = +14$ kN

zugeh. $w_2 = \frac{14}{10} \cdot 0{,}02 = 0{,}028$ m

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2. Iteration:

$$-V_1 \cdot 3{,}0 - 300 \cdot 0{,}028 \overset{!}{=} 0 \quad \rightsquigarrow \quad V_1 = -2{,}8\,\text{kN}$$

$$V_1 = -2{,}8 = V_3 \cdot [\text{kN}]$$

$$V_4 = 10 + 2{,}8 + 2{,}8 = 15{,}6\,\text{kN}$$

Zugeh. $w_2 = \frac{15{,}6}{10} \cdot 0{,}02 \approx 0{,}032\,\text{m}$

Gas geben!

$0{,}0312$

3. Iteration: $-V_1 \cdot 3{,}0 - 300 \cdot 0{,}033 \overset{!}{=} 0 \quad \rightsquigarrow \quad V_1 = -3{,}3\,\text{kN}$

$$V_3 = V_1 = -3{,}3\,\text{kN}$$

$$V_4 = 10 + 3{,}3 + 3{,}3 = 16{,}6\,\text{kN}$$

Zugeh. $w_2 = \frac{16{,}6}{10} \cdot 0{,}02 = 0{,}0332$

analytisch: $w_3 = 0{,}033\,\text{m}$

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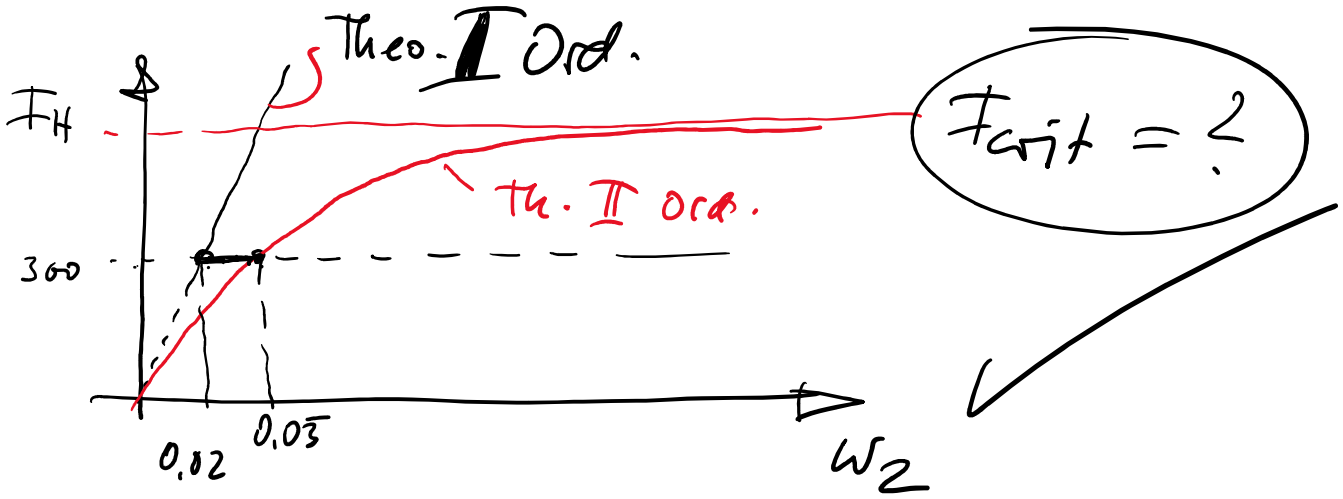
Zustandsgr. nach Th. II. Ord

$V_1 = V_3 = -3{,}33\ kN$; $V_4 = 16{,}66\ kN$

$H_A = 300\ kN$; $w_2 = 0{,}03\overline{3}\ m$

$N_{42} = +16{,}66\ kN$; $N_{12} = N_{23} = -\sqrt{300^2 + 3{,}3^2}$

$= -300{,}02\ kN$



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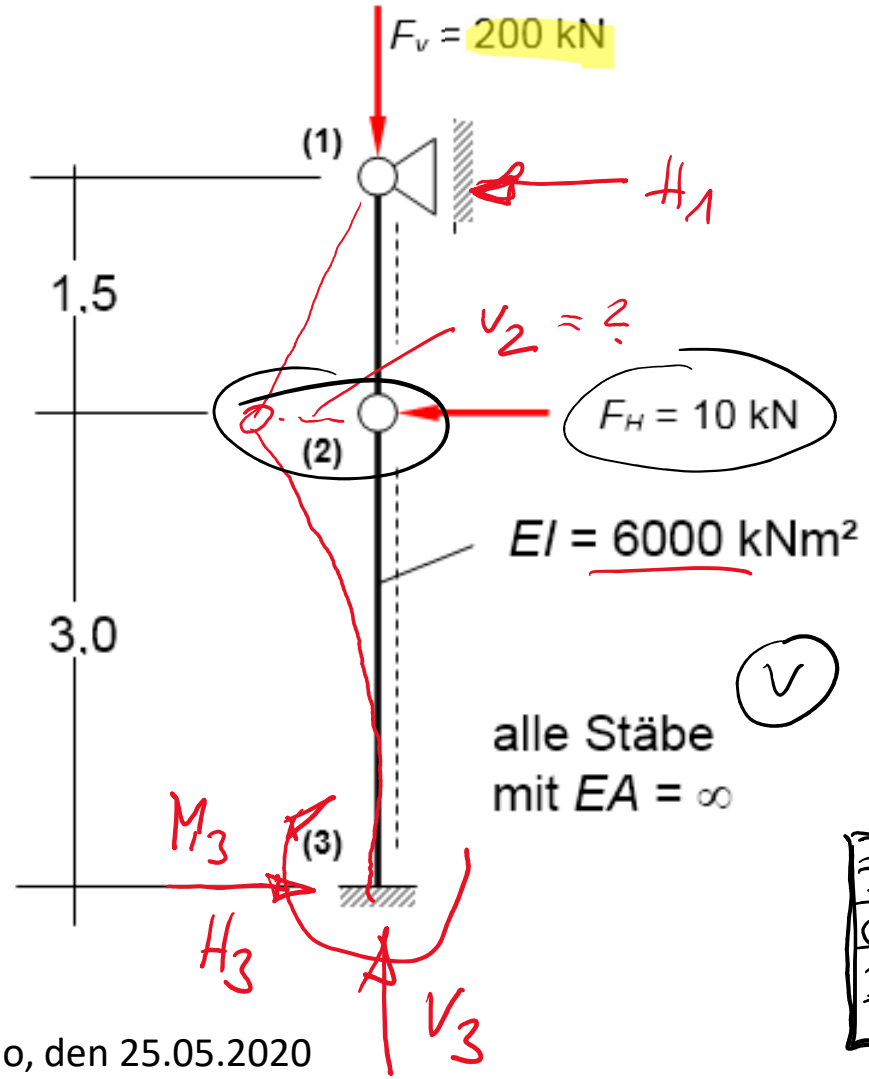
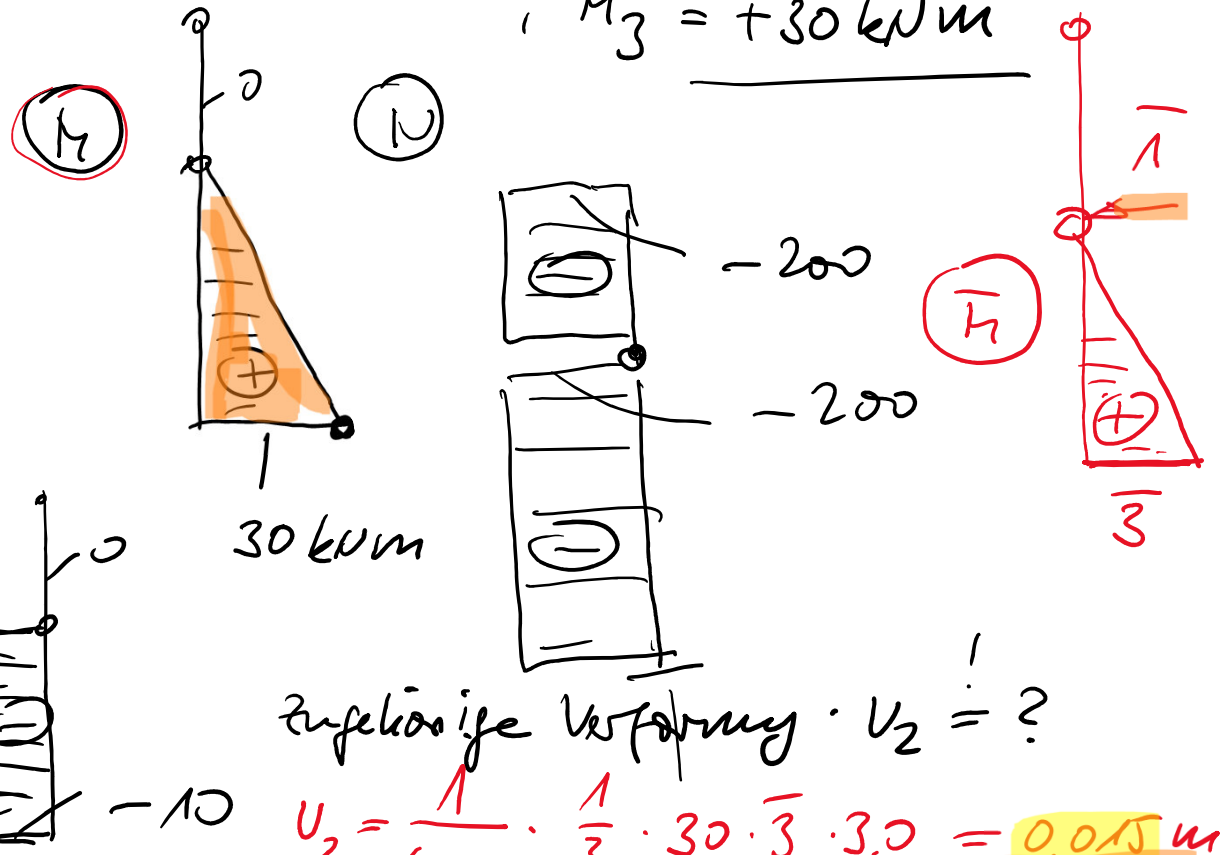
(C)



Zustandsgr. nach Th. I. O.

V_3 = 200 kN ; H_3 = 10 kN

; M_3 = +30 kNm



Zugehörige Verformung · v_2 = ?

$$v_2 = \frac{1}{6000} \cdot \frac{1}{3} \cdot 30 \cdot \bar{3} \cdot 3{,}0 = 0{,}015\ \text{m}$$

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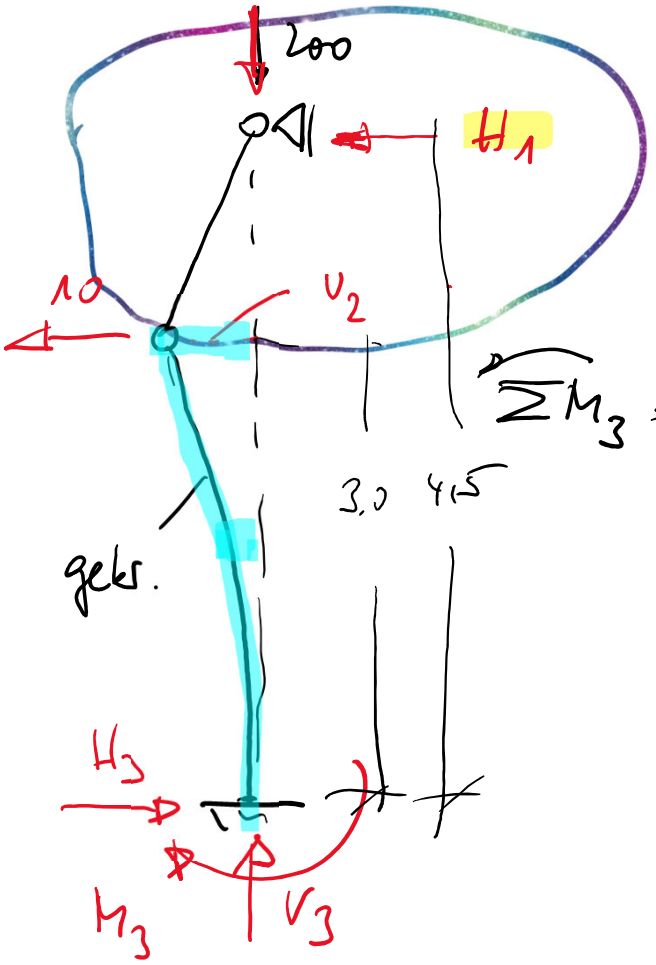


$\curvearrowright \Sigma M_{2,OTS} \stackrel{!}{=} 0$: 1. Iteration $-200 \cdot U_2 + H_1 \cdot 1{,}5 \stackrel{!}{=} 0$

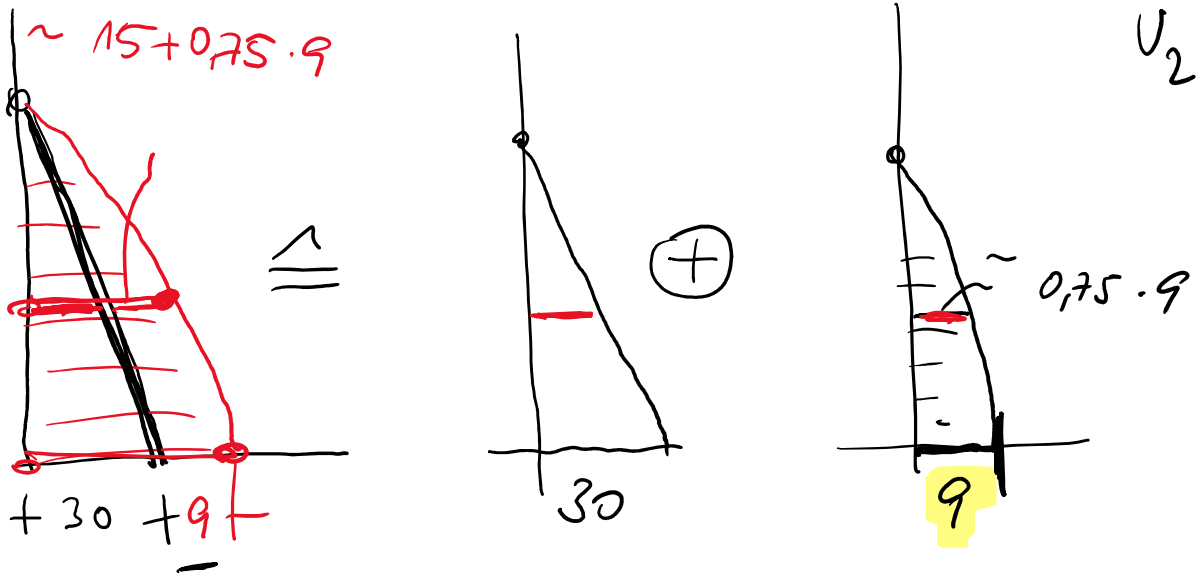
$\rightsquigarrow \underline{H_1 = 2{,}0\ kN}$

$\rightsquigarrow H_3 = 10 + 2{,}0 = \underline{12\ kN}$; $V_3 = \underline{200\ kN}$

$\curvearrowright \Sigma M_3 \stackrel{!}{=} 0$: $-M_3 + 10 \cdot 3 + 2 \cdot 4{,}5 = 0$

$\rightsquigarrow \underline{M_3 = 39\ kNm}$

zugeh. Wegfgr. $U_2 = ?$



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$$U_2 = \underbrace{\frac{1}{6000} \cdot \frac{1}{3} \cdot 30 \cdot \bar{3} \cdot 3.0}_{\text{30} \quad \bar{3}} + \frac{1}{6000} \cdot \frac{5}{12} \cdot 9 \cdot \bar{3} \cdot 3.0 = 0.021\,m$$

2. Hauptsatz:

$$-200 \cdot 0.021 + H_1 \cdot 1.5 = 0 \rightsquigarrow H_1 = 2.8\,kN$$

$$\rightsquigarrow H_3 = 10 + 2.8\,kN = 12.8\,kN \; ; \; V_3 = 200\,kN$$

$\Sigma M_3 \overset{!}{=} 0$: $+10 \cdot 3 + 2.8 \cdot 4.5 - M_3 = 0$

$$M_3 = 42.6\,kNm \quad (30 + 12.6) \quad \Delta M$$

$$U_{2,neu} = \frac{1}{6000} \cdot 90 + \frac{1}{6000} \cdot \frac{5}{12} \cdot 12.6 \cdot \bar{3} \cdot 3 = 0.023\,m$$

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3. Iteration

$-200 \cdot 0{,}023 + H_1 \cdot 1{,}5 \overset{!}{=} 0 \quad \rightsquigarrow \quad H_1 = 3{,}07\ \text{kN}$

$\rightsquigarrow \quad H_3 = 13{,}07\ \text{kN} \quad ; \quad V_3 = 200\ \text{kN}$

$\sum M_3 \overset{!}{=} 0: \quad 10 \cdot 3{,}0 + 3{,}07 \cdot 4{,}5 - M_3 \overset{!}{=} 0$

$\rightsquigarrow \quad M_3 = 30 + 13{,}82 = 43{,}82\ \text{kNm}$

(ΔM_{II})

Zugehörige Verformung: $v_2 = \frac{1}{6000} \cdot 90 + \frac{1}{6000} \cdot \frac{5}{12} \cdot 13{,}82 \cdot 3 \cdot 3$

$= 0{,}024\ \text{m}$ ✓

(M) - Verlauf ; (V) - Verlauf ; (w) - Verlauf

(A) - frei $\rightsquigarrow$ Selber machen ✓

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