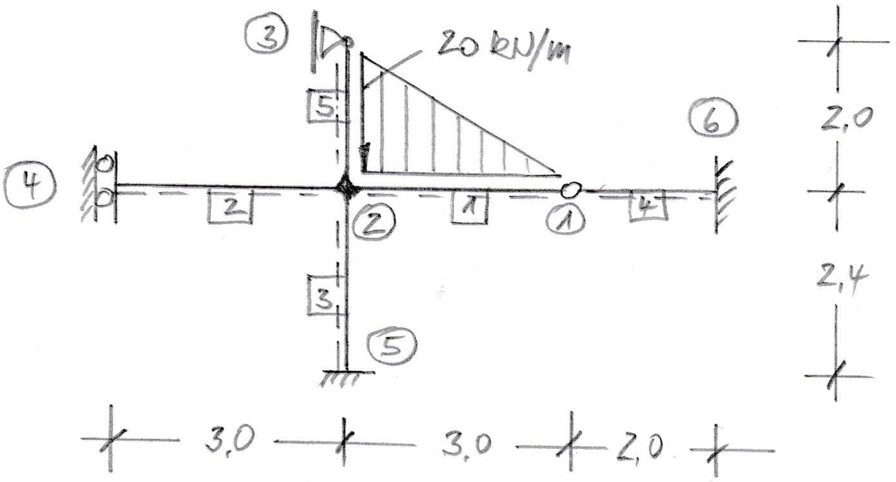
Übungsaufgabe 7: Federmodell + WGV in Matrizenverf.

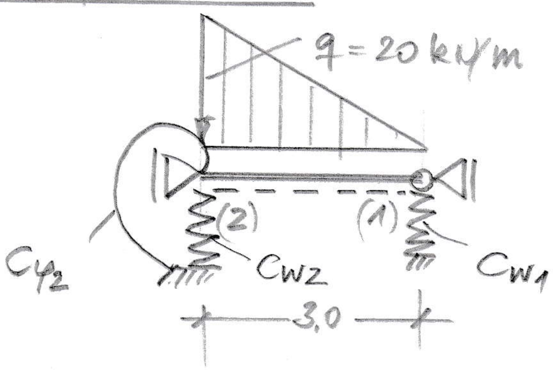
– System:



Steifigkeiten:

i	EA	EI
1	30000	3000
2	∞	4000
3	40000	3000
4	∞	2000
5	12345	2400

– Ersatzfedermodell:



Federsteifigkeiten:

$$c_{\varphi_2} = c_{\varphi_{2,4-2}} + c_{\varphi_{2,3-2}} + c_{\varphi_{2,5-2}}$$

$$= \frac{1EI}{l} + \frac{3EI}{l} + \frac{4EI}{l}$$

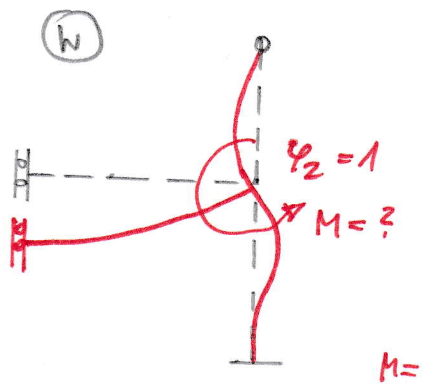
$$= \frac{4000}{3} + \frac{3\cdot 2400}{2} + \frac{4\cdot 3000}{2{,}4}$$

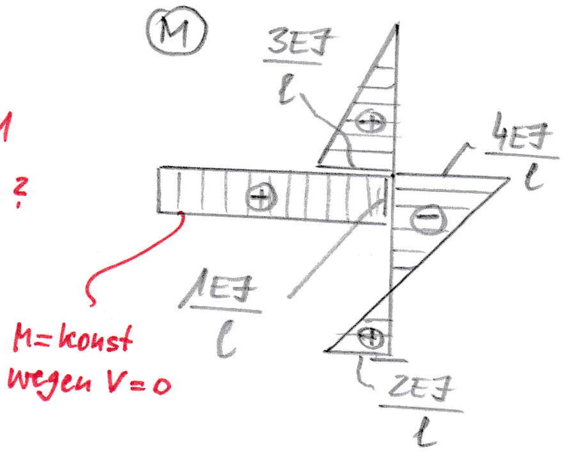
$$= \underline{\underline{9933{,}3\ \mathrm{kNm/rad}}}$$

$$c_{w1} = \frac{3EI}{l^3} = \frac{3\cdot 2000}{2{,}0^3} = \underline{\underline{750\ \mathrm{kN/m}}}$$

$$c_{w2} = \frac{EA}{l} = \frac{40000}{2{,}4} = \underline{\underline{16666{,}\bar{6}\ \mathrm{kN/m}}}$$

Erklärung zu c_{φ_2}:



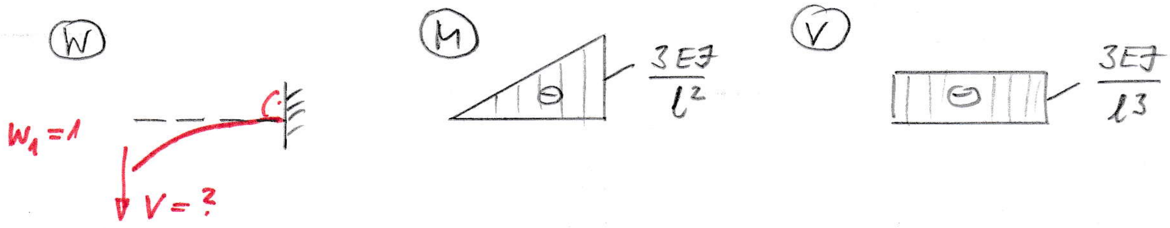


alle 3 „Federn" wirken auf den gleichen Drehwinkel

⇝ Parallelschaltung

$$C = C_1 + C_2 + C_3$$

Erklärung zu c_{w1}:



– Berechnung des Ersatzmodells mit Hilfe des UGV in M.-darst.

Tipp: Wenn es mit dem Excel-Tool nachgerechnet werden soll, Stab 1 in 3 Stäbe (jeweils $l = 1\,m$) aufteilen und rechnen lassen!

Inzidenztafel:

i	a	e	l	β	EA	EJ	qa	qe
1	2	1	3,0	0°	30000	3000	20,0	0,0

– Schritt 1: Erstellen der Stabsteifigkeitsmatrix und des zugeh. Lastvektors (lokal = global)

$$\bar{s}^1 = \bar{K}^1 \cdot \bar{v}^1 + \bar{s}^{10}$$

$$\begin{array}{c} \bar{s}_2^1 \\ \left[\begin{array}{c} N_2^1 \\ V_2^1 \\ M_2^1 \\ \hline N_1^1 \\ V_1^1 \\ M_1^1 \end{array}\right] \\ \bar{s}_1^1 \end{array} = \begin{array}{c} \bar{K}_{22}^1 \qquad\qquad \bar{K}_{21}^1 \\ \left[\begin{array}{ccc|ccc} 10000 & 0 & 0 & -10000 & 0 & 0 \\ 0 & 1333,\bar{3} & -2000 & 0 & -1333,\bar{3} & -2000 \\ 0 & -2000 & 4000 & 0 & 2000 & 2000 \\ \hline -10000 & 0 & 0 & 10000 & 0 & 0 \\ 0 & -1333,\bar{3} & 2000 & 0 & 1333,\bar{3} & 2000 \\ 0 & -2000 & 2000 & 0 & 2000 & 4000 \end{array}\right] \\ \bar{K}_{12}^1 \qquad\qquad \bar{K}_{11}^1 \end{array} \cdot \begin{array}{c} \bar{v}_2^1 \overset{!}{=} v_2 \\ \left[\begin{array}{c} u_2^1 \\ w_2^1 \\ \varphi_2^1 \\ \hline u_1^1 \\ w_1^1 \\ \varphi_1^1 \end{array}\right] \\ \bar{v}_1^1 \overset{!}{=} v_1 \end{array} + \begin{array}{c} \bar{s}_2^{10} \\ \left[\begin{array}{c} 0,0 \\ -21,0 \\ +9,0 \\ \hline 0,0 \\ -9,0 \\ -6,0 \end{array}\right] \\ \bar{s}_1^{10} \end{array}$$

– Schritt 2: Gesamtsystem und Gleichgewichtsbed.

$$\underline{\bar{K}}_{G,C} = \underline{\bar{K}}_G + \underline{\bar{C}}_G$$

$$\begin{array}{c c} & \begin{array}{ccc|ccc} & 1 & & & 2 & \end{array} \\ \begin{array}{c} \\ 1 \\ \\ \\ 2 \\ \\ \end{array} & \left[\begin{array}{ccc|ccc} 10000 & 0 & 0 & -10000 & 0 & 0 \\ 0 & 1333{,}3 + 750 & 2000 & 0 & -1333{,}\bar{3} & 2000 \\ 0 & 2000 & 4000 & 0 & -2000 & 2000 \\ \hline -10000 & 0 & 0 & 10000 & 0 & 0 \\ 0 & -1333{,}\bar{3} & -2000 & 0 & 1333{,}3 + 16666{,}\bar{6} & -2000 \\ 0 & 2000 & 2000 & 0 & -2000 & 4000 + 9933{,}\bar{3} \end{array}\right] \end{array} \cdot \underbrace{\left[\begin{array}{c} \bar{u}_1 \\ \bar{w}_1 \\ \bar{\varphi}_1 \\ \hline \bar{u}_2 \\ \bar{w}_2 \\ \bar{\varphi}_2 \end{array}\right]}_{\underline{\bar{V}}_G} + \underbrace{\left[\begin{array}{c} 0{,}0 \\ -9{,}0 \\ -6{,}0 \\ \hline 0{,}0 \\ -21{,}0 \\ +9{,}0 \end{array}\right]}_{-\underline{\bar{r}}_G} = 0$$

– Schritt 3: Einbau der Randbedingungen

$$\underline{\bar{K}}_{G,C,RB}: \quad \begin{array}{c c} & \begin{array}{ccc|ccc} & 1 & & & 2 & \end{array} \\ \begin{array}{c} \\ 1 \\ \\ \\ 2 \\ \\ \end{array} & \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2083{,}\bar{3} & 2000 & 0 & -1333{,}\bar{3} & 2000 \\ 0 & 2000 & 4000 & 0 & -2000 & 2000 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1333{,}3 & -2000 & 0 & 18000 & -2000 \\ 0 & 2000 & 2000 & 0 & -2000 & 13933{,}\bar{3} \end{array}\right] \end{array} \cdot \underbrace{\left[\begin{array}{c} \bar{u}_1 \\ \bar{w}_1 \\ \bar{\varphi}_1 \\ \hline \bar{u}_2 \\ \bar{w}_2 \\ \bar{\varphi}_2 \end{array}\right]}_{\underline{\bar{V}}_G} = \underbrace{\left[\begin{array}{c} 0 \\ 9{,}0 \\ 6.0 \\ \hline 0 \\ 21{,}0 \\ -9{,}0 \end{array}\right]}_{+\underline{\bar{r}}_{G,RB}}$$

– Schritt 4: Lösung des Gleichungssystems

$$\underline{\bar{V}}_G = \left[\begin{array}{c} 0{,}0 \\ 7{,}261 \cdot 10^{-3} \\ -7{,}064 \cdot 10^{-4} \\ \hline 0{,}0 \\ 1{,}473 \cdot 10^{-3} \\ 1{,}375 \cdot 10^{-3} \end{array}\right]$$

$\underline{\bar{V}}_1 \overset{!}{=} \underline{\bar{V}}_1^1 = \underline{\bar{V}}_e^1 = \underline{V}_e^1$!

$\underline{\bar{V}}_2 \overset{!}{=} \underline{\bar{V}}_2^1 = \underline{\bar{V}}_a^1 = \underline{V}_a^1$!

– **Schritt 5 : Nachlaufrechnung**

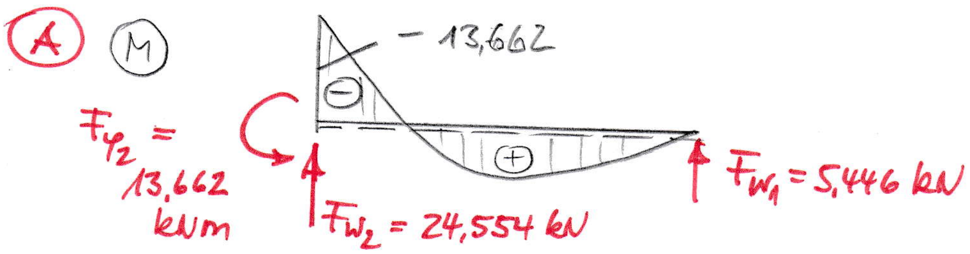
Einsetzen der zugeordn. Knotenweggrößen (≙ Stabendweggrößen) in Gleichung nach Schritt 1

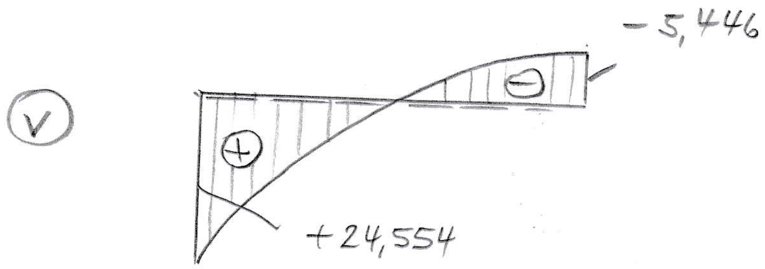
$$\underline{S}^1 = \underline{K}^1 \cdot \underline{V}^1 + \underline{S}^{10} = \underline{S}^1$$

$$\begin{array}{c} \\ \end{array}\begin{bmatrix} N_2 \\ V_2 \\ M_2 \\ \hline N_1 \\ V_1 \\ M_1 \end{bmatrix} = \left[\begin{array}{ccc|ccc} 10000 & 0 & 0 & -10000 & 0 & 0 \\ 0 & 1333,\overline{3} & -2000 & 0 & -1333,\overline{3} & -2000 \\ 0 & -2000 & 4000 & 0 & 2000 & 2000 \\ \hline -10000 & 0 & 0 & 10000 & 0 & 0 \\ 0 & -1333,\overline{3} & 2000 & 0 & 1333,\overline{3} & 2000 \\ 0 & -2000 & 2000 & 0 & 2000 & 4000 \end{array}\right] \cdot \begin{bmatrix} 0,0 \\ 1,473 \cdot 10^{-3} \\ -1,375 \cdot 10^{-3} \\ \hline 0,0 \\ 7,261 \cdot 10^{-3} \\ -7,064 \cdot 10^{-4} \end{bmatrix} + \begin{bmatrix} 0,0 \\ -21,0 \\ +9,0 \\ \hline 0,0 \\ -9,0 \\ -6,0 \end{bmatrix} = \begin{bmatrix} 0,0 \\ -24,6 \\ 13,7 \\ \hline 0,0 \\ -5,4 \\ 0,0 \end{bmatrix}$$

(gerundet)

Darstellung der Schnittgrößen am Ersatzmodell :





Darstellung der Schnittgrößen am ursprüngl. System :

Aufteilung des Einspannmomentes auf die Stäbe 3, 2 und 5

$$M_{2,\ell} = -13,662 \cdot \frac{1333,\overline{3}}{9933,\overline{3}} = \underline{-1,834 \text{ kNm}} \mathrel{\hat{=}} \frac{1EJ}{\ell} \cdot \varphi_2$$

$$= \frac{1 \cdot 4000}{3} \left(-1,375 \cdot 10^{-3}\right)$$

$$= \underline{\underline{-1,834 \text{kNm}}} \quad ✓$$

$$M_{2,o} = -13{,}662 \cdot \frac{5000{,}0}{9933{,}\overline{3}} = \underline{-6{,}877\ kNm} \;\hat{=}\; \frac{4EJ}{l}\cdot\varphi_2$$

$$= \frac{4\cdot 3000}{2{,}4}\cdot(-1{,}375\cdot 10^{-3})$$

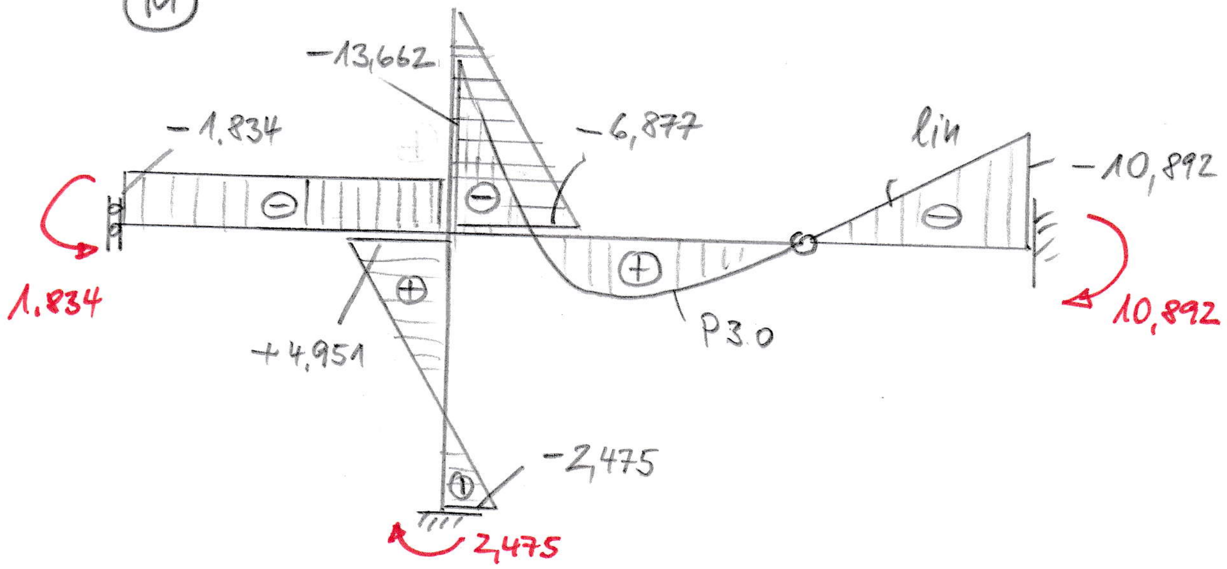
$$= \underline{-6{,}877\ kNm}$$

$$-M_{2,u} = -13{,}662\ \frac{3600}{9933{,}3} = \underline{-4{,}951\ kNm} \;\hat{=}\; \frac{3EJ}{l}\cdot\varphi_2$$

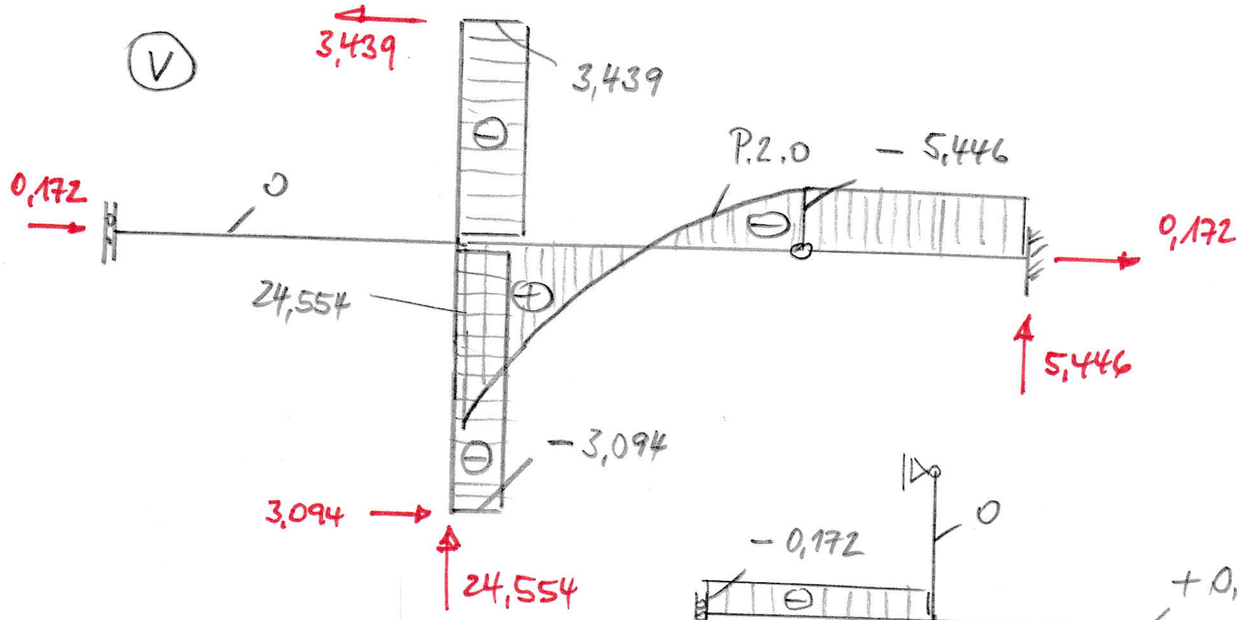
$$= \frac{3\cdot 2400}{2{,}0}(-1{,}375\cdot 10^{-3})$$

$$= \underline{-4{,}951\ kNm}$$

M



V



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